tangent depth analysis (1) Basics

The fundamental equation in IIT:

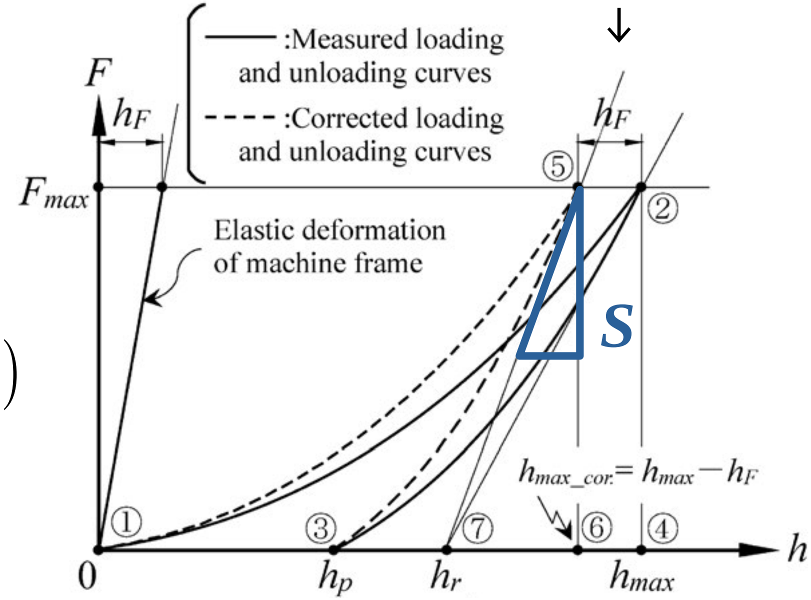
$$E_r = \frac{\sqrt{\pi}}{2} \cdot \frac{S}{\sqrt{A}}$$

$$\frac{1}{E_r} = f(E) = \frac{1-\nu_i^2}{E_i} + \frac{1-\nu_s^2}{E_s} = I(E) + S(E)$$

E_r: Reduced modulus

E, ν: Young's modulus, Poisson's ratio

C_F calibrated curve ↓



tangent depth h_r is not affected by C_F

$$S = \frac{F_{max}}{(h_{max\_cor.} - h_r)} = \frac{F_{max}}{(h_{max} - F_{max} \cdot C_F - h_r)}$$

S: unloading slope of flame compliance(C_F) calibrated curve (above).

A: (projected area of contact) → a function of h_r in tangent depth analysis

The fundamental equation can be shown as follows.

$$E_r = \frac{\sqrt{\pi}}{(2\sqrt{A(h_r)})} \frac{F_{max}}{(h_{max} - F_{max} \cdot C_F - h_r)}$$

tangent depth analysis - (2) calculation of C_F

Young's modulus and hardness of standard specimens (BK7, Fused silica) are assumed to be constant.

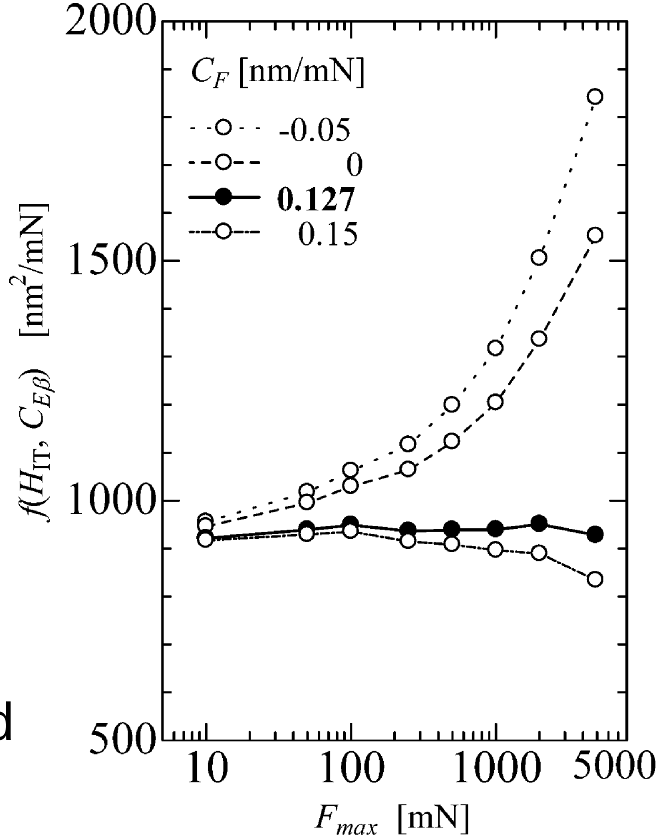
$$E_r = \frac{\sqrt{\pi}}{\left(2\sqrt{A(h_r)}\right)} \frac{F_{max}}{\left(h_{max} - F_{max} \cdot C_F - h_r\right)} = const.$$

$$H_{IT} = \frac{F_{max}}{A(h_r)} = const.$$

Therefore， following f without $A(h_r)$ should be constant.

$$f(H_{IT}, C_{E\beta}) \equiv F_{max} \cdot \left(\frac{h_{max} - h_r}{F_{max}} - C_F\right)^2 = \frac{\pi H_{IT}}{4E_r^2} = const.$$

The value of C_F, which makes f constant, is calculated and used. Note that values of Young's modulus and hardness are not necessary in this step.



(a) BK7

1) T. ISHIBASHI, Y. YOSHIKAWA, M. YAMAMOTO, K. MIYAHARA, T. YAMAMOTO, M. OHKI, S. KATAYAMA: J. Mater. Test. Res. Vol.62, 146 (2017)

tangent depth analysis – (3) area function

In order to obtain the area function $A(h_r)$, a standard specimen with known Young's modulus and Poisson's ratio is necessary. (BK7 or Fused silica, etc.)

$$E_r = \frac{\sqrt{\pi}}{\left(2\sqrt{A(h_r)}\right)} \frac{F_{max}}{\left(h_{max} - F_{max} \cdot C_F - h_r\right)}$$

The above fundamental equation can be deformed as follows.

$$\sqrt{A(h_r)} = \frac{\sqrt{\pi}}{(2 \cdot E_r)} \frac{F_{max}}{\left(h_{max} - F_{max} \cdot C_F - h_r\right)}$$

This means one curve of standard specimen gives a specific point of $A(h_r)$ at h_r.
→$A(h_r)$ can be fitted to an appropriate function with enough number of curves.

$$\sqrt{A(h_r)} = \frac{\exp\left(a \cdot \{\log_e(C_1 \cdot h_r)\}^b + c\right)}{C_2}$$

C_1,C_2 : fixed constants a, b, c: fitting parameters

Once $A(h_r)$ is obtained, Young's modulus E_r and indentation hardness H_{IT} can be calculated for any specimens. (Poisson's ratio is necessary for Young's modulus.)